Evolutionary Topological Optimum Design

Project for EvoNet Summer School 2001,
Thessaloniki, Greece

M. Schoenauer,
Équipe Évolution Artificielle et Apprentissage
CMAP (UMR CNRS 7641)
École Polytechnique – Palaiseau
Marc.Schoenauer@polytechnique.fr
http://www.cmap.polytechnique.fr/~marc

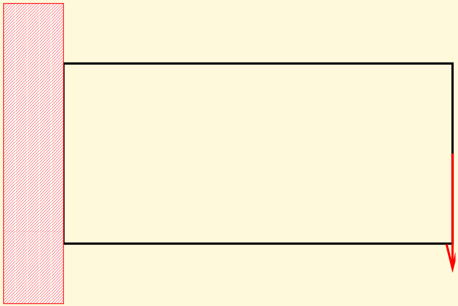
1. Optimum Design

- Find the shape of a structure
- inside a given design domain
- with constraints on the mechanical behavior

Stiffness optimization
Bounds on the displacement when a given load is applied
Modal Optimization
Avoid undesirable eigenfrequencies

e.g. the sea-sick generator 1Hz in cars

1.1. Test problem: the 2D cantilever plate



Problem

$$\begin{aligned} \min \quad & Weight \\ \text{with constraints} \quad & D_{load} < D_{lim} \end{aligned} \tag{1}$$

D_{load}: displacement of the structure when the load is applied
D_{lim} prescribed limit on that displacement

Simple mechanical model Linear elasticity, 2D model

1.2. Brief State of the Art

Shape optimization:

gradually and slowly modify a given shape

- allows one to use gradient methods
- local method, numerically unstable
- cannot modify the **topology** of the starting shape

We are interested in Topological Optimum Design

Homogenization: to-date method for TOD

- Considers a relaxed problem

  looks for a continuous density $\in [0, 1]$)

- allows one to use gradient methods

- numerically very robust

Limitations:

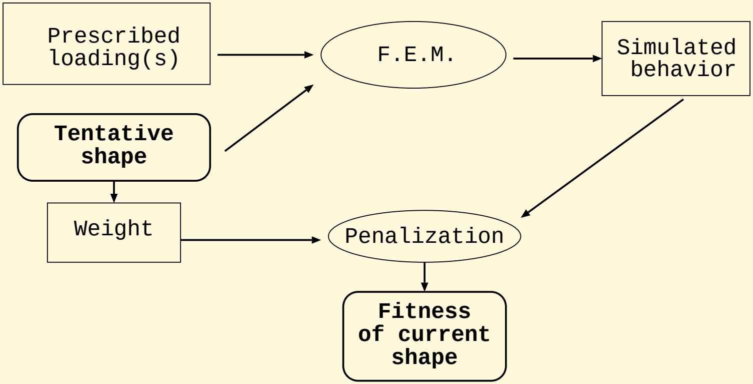
- **Mechanical model** Limited to linear elasticity
- Modal optimization?
- Load on the unknown boundary? e.g. uniform pressure
- Always find the same solution for a given problem

  No **creativity** to hope for

$\longrightarrow$ **Evolutionary Algorithms**.

2. Evolutionary approach

2.1. A tentative fitness



- What fitness?
- What representation?

2.2. Penalized fitness

$$(Weight + \alpha(D_{load} - D_{lim})^+ \tag{2}$$

x^+ = positive part of x

or, in case of multiple loadings

$$(Weight + \alpha^i(D^i_{load} - D^i_{lim})^+ \tag{3}$$

D^i_{load} displacement of the structure under load i
D^i_{lim} given limit on that displacement

of course, one FE analysis is needed for each load!

3. The representation problem

- Phenotype $\equiv$ 2-partitions of the design domain Not any partition (e.g. continuous boundary)
- What genotype ?

and variation operators?

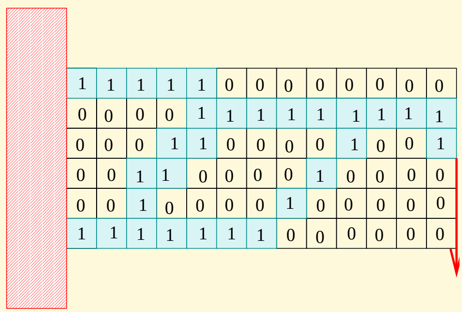
Remember that EAs are flexible

able to work in weird search spaces!

3.1. Bitarrays

- Use the mesh of the FE analyses
- “Natural” from FE point of view
- Used in all early works

Jensen 92, Chapman & Jakiela 94-95, Kane 96



The **complexity** of the representation (number of “variables”) is that of the mesh

3.2. Bitarray: Results

Limits of homogenization method overcome:

- Any mechanical model
- Multiple quasi-optimal solutions
- Loading on the unknown boundary (e.g. external pressure).

BUT

- the complexity of the representation is that of the mesh.
- accurate FEM require refined mesh — not to mention 3D!
- population size $\propto$ individual size — Goldberg & al. 92, Cerf 95

$\longrightarrow$ **Need for mesh-independent representations**

3.3. Voronoi representation

Voronoi diagrams

Given a set of *sites* $S_1, \ldots, S_n$ in $\mathbb{R}^2$, define

$$Cell(S_i) = \{M; d(M, S_i) = Min_j d(M, S_j)\}$$

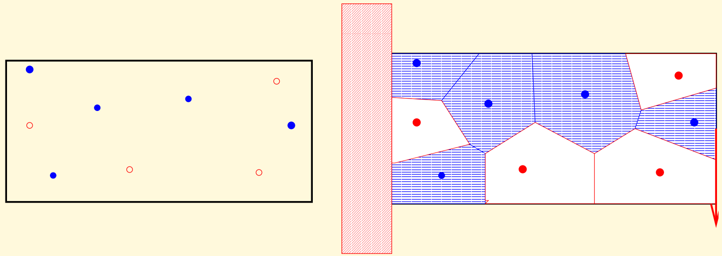
$Cell(S_i)$ is the set of points whose nearest site is S_i

$\longrightarrow$ A partition of the space by convex polygons.

Shape representation

Each site S_i is given a label (0/1, for void/material).

Each cell is labeled as its corresponding site:



Genotype:

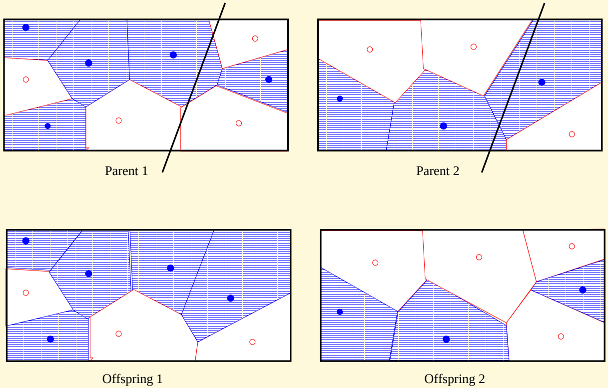
An individual is a list of cells.

$(n, (S_1, c_1), \ldots, (S_n, c_n))$, $1 \leq n \leq NMAX$, $c_i \in \{0, 1\}$

Variation operators

RECOMBINATION

Geometric exchange of sites.



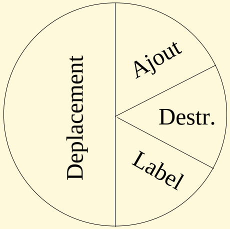
Mutations

- Gaussian displacement of a site

room for self-adaptation

- Change of label of a site.
- Creation of a site.
- Destruction of a site.

The choice of the mutation is governed by user-supplied weights.



3.4. Other representations

- Holes representation
  "dig holes" in a full plate
  genotype is a variable-length list of elementary shapes
- Voronoi bars
  Voronoi cells are truss elements
- IFS representation
  The shape is the attractor of an Iterated Function System

4. SummerSchool Project

4.1. Available tools

- A full Voronoi-based EA — written in EO library
- A lab-made FEM solver — interfaced with EO
- An ad hoc graphical output

Fitness computation

- FEAs are CPU costly — 3mn/run for a 10×10 mesh
  > 1 day for a 100×10 mesh
- Dummy fitness available — Comparison with a target shape

4.2. Project directions

Up to student preferences

On the representation side: Multi-holes representation

- Holes with multiple elementary shapes
- Comparison with Voronoi
- Simultaneous evolution of “material” and “void” holes

Ultimately: design and implement a modular representation

with reusable modules

using GP, or more specific approaches (e.g. quad-trees)

On the fitness side

- Links between representation and fitness

  shape of target for the dummy fitness

  size of the mesh for the TOD fitness

- Implementation of multi-objective approach

  The problem is in fact multi-objective

  Building blocks available in EO library

- Comparison constraint vs multi-objective e.g. in the case of multiple loads

- Complexity viewed as another objective